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Stochastic Mechanics of Soil Erosion

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Résumé

Une nouvelle méthode stochastique est proposée pour estimer l'érosion du sol. Cette méthode est basée sur un calcul de probabilité de l'excès des forces de déplacement sur les forces stabilisantes. La méthode permet d´établir théoriquement les corrélations entre l'intensité de l'érosion et les facteurs la controlant.

Introduction

Soil erosion mechanics is determined by complex interaction between turbulent flow and structured soil. This interaction is stochastic in nature, due to the complexity and randomness of geo-mechanical structural and electro-chemical forces in soil, as well as the velocity fluctuations in shallow overland flow with rough boundaries. Soil erosion occurs when and where driving hydrodynamic forces exceed geo-mechanical and gravitational resisting forces. The original idea of stochastic modelling of erosion was formulated and used by H.A. Einstein (1937) for modelling erosion of non-cohesive sediments. There have been already several attempts (Nearing, 1991; Wilson, 1993; Sidorchuk, 2001) that incorporate the probability of soil particle detachment being related to the excess of driving forces over stabilising forces, and the rate of detachment being related to the frequency of flow velocity fluctuations. Understanding of soil erosion stochastic mechanics leads to the formulation of third generation of soil erosion models.

Methods

Soil erosion E is the rate of lowering of soil surface Z_0. This rate varies in different points of soil surface at a given time, and changes through time in a given point. Soil erosion rate is spatial (on the area S) and temporal (during some period T) averaging of the rates of local erosion

$$E = \frac{1}{ST}\iint_{ST} \frac{\partial Z_0}{\partial t}\, ds\, dt \qquad (1)$$

Local erosion of cohesive soil is a type of rather slow erosion, when water mainly contacts and affects a surface layer of soil. Local erosion is discrete: the surface diminishes when the soil fragment is detached and is stable between detachment events. Within an event of detachment local erosion rate is equal to the ratio of detached fragment thickness D and detachment duration τ_d. In the periods between detachment events, the local erosion rate is zero.

The soil surface is stable when and where driving forces F_{dr} are equal to resisting forces F_{res} and the resultant force Θ is equal to zero. The soil fragment becomes unstable and its movement begins when and where driving forces F_{dr} exceed resisting forces F_{res}:

$$\Theta = \left|F_{dr} - F_{res}\right| > 0 \qquad (2)$$

The acceleration of such an unstable fragment within the soil surface layer along the axis Z in a direction of resultant force Θ is described with the second Newton law:

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$$V_a \quad _s \frac{d^2 Z}{dt^2} = \qquad (3)$$

Here $_s$ is soil aggregate density and V_a is its volume; t is time. The process of soil fragment detachment takes some period of time τ_d, which can be calculated with (3). Let us define soil fragment as detached when all it's cohesive links are broken. This condition is usually achieved when the soil fragment is removed from its initial position to a distance equal to its height D. Then, for the simplest case Θ independent of Z, period of detachment equals to

$$_d = \sqrt{\frac{2 \quad _s V_a D}{}} \qquad (4)$$

Local erosion rate can be calculated with

$$\frac{Z_0}{t} = \sqrt{\frac{D}{2 \quad _s V_a}} \qquad (5)$$

Eqn 5 is valid both at the periods of destabilisation of soil surface and at the periods of stable surface, when local erosion rate is zero. It can therefore it can be used to calculate averaged erosion rate for the whole area S and continuous time period T

$$E = \frac{1}{ST} \ _{ST} \sqrt{\frac{D}{2 \quad _s V_a}} dsd \qquad (6)$$

Eqn 6 is a formula for the calculation of mean value of variable $= \sqrt{\frac{D}{2 \quad _s V_a}}$. It can be written in the form common for probability theory

$$E = \quad p \ d \qquad (7)$$

where p_α is the probability density function for α. The representation of erosion rate in the form of Eqn 7 opens the possibility of using the results of probability theory for the theoretical determination of erosion. The main question raised is the determination of local erosion rate α and its probability density from the stochastic characteristics of water flow and soil structure. These characteristics can be estimated with two main approaches. Within the first (discrete) approach soil structure is considered as a combination of solid aggregates connected with cohesive resistance forces. Hydrodynamic driving forces acting on the soil surface are included implicitly by parameterisation of flow velocity and pressure, depending on geometry of soil aggregates and soil surface. These forces are drag (F_{fd}) and lift force (F_l).

$$F_{fd} = C_R \quad S_d \frac{U^2}{2} \qquad (8)$$

$$F_l = C_y \quad S_a \frac{U^2}{2} \qquad (9)$$

The main resistance forces are the normal component of submerged weight (F_{wn}) and geo-mechanical force of cohesion (F_c).

$$F_{wn} = gV_a (\quad _s \quad) \qquad (10)$$

$$F_c = C_0 S_b \qquad (11)$$

Here C_R is the coefficient of drag resistance; C_y is the coefficient of uplift; U is the actual near-bed flow velocity; $_s$ and are the soil aggregate density and water density, respectively; C_0 is soil cohesion. S_d is the area of soil aggregate cross-section perpendicular to flow; S_a is the cross-

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section area of the soil aggregate parallel to the flow (vertical projection, equal to V_a/D); S_b is the area of the soil aggregate surface where it is strongly attached to surrounding soil aggregates. Cohesion between aggregates occurs only on such contact surfaces, therefore the ratio $I_s=S_b/S_a$ characterises soil integrity, which is very important for aggregated soil with micro-cracks. Within first approach, the rate of local erosion is

$$= \sqrt{C_R k_d \frac{}{_s} \frac{U^2}{4} + C_y \frac{}{_s} \frac{U^2}{4} \quad g \frac{_s}{2 \; _s} D \quad C_0 I_s} \quad (12)$$

Here k_d is S_d/S_a. If expression under square root become zero or negative, then $\alpha=0$. For this case Eqn. 12 is a generalized form of well-known expression for incipient motion criteria calculation. Probability density function (PDF) for α can be calculated, if PDFs for each variable in Eqn. 12 is known.

To derive parameterisations for flow properties such as velocity distributions in shallow overland flows the writers used both laboratory experiments and numerical simulations. The laboratory experiments have been conducted using a specially designed overland flow flume where the flow properties were investigated for both sub- and super-critical flows over flat, smooth surfaces and over surfaces with roughness elements modelled with emergent cylinders covering a range of diameter-to-depth ratio values.

For studying overland flows numerically Large Eddy Simulation has proved to be the most suitable tool for erosion modelling, since it can accommodate complex solid boundaries adequately, gives full distributions of velocity and pressure, and is compatible with a variety of methods for incorporating sediment dynamics. This modelling method can provide extensive information on velocity and sediment fields that are needed to underpin the stochastic concept. Parameterisations for soil structure parameters have been developed based on specially designed laboratory experiments and following the theoretical methodology proposed by Kolmogorov. The Kolmogorov-type algorithm of soil particles failure can be implemented numerically. Experiments show that normal distribution can be used for flow velocity U, and logarithmically normal distribution for soil aggregate size D, cohesion C_0 and integrity I_s. PDF for their function α is difficult to express analytically because numerical estimates show complicated relationship between p_α and the main factors of erosion.

In the second (continual) approach, soil is regarded as a continuum, described with changes in time and space surface geometry and critical resistance forces for a given soil volume. Hydrodynamic driving forces are presented explicitly through hydrodynamic pressure gradients at soil surface irregularities. The soil erosion rate is then theoretically defined in terms of probability density functions for soil resistance forces and pressure gradients on the soil surface irregularities. The first (discrete) approach is more suitable in arid and semi-arid environments, and the second (continual) approach is better for soils in humid and submerged conditions.

Results

Numerical experiments within the first approach were carried out to illustrate the influence of several stochastic variables on the erosion rate. The appointed range of the mean near-bed flow velocity U was 0.1–2.2 m/s, that of the mean cohesion C_0 was 1–40 kPa, the mean soil integrity I_s ranged from 0.1 to 0.5, and the mean size D of a soil aggregate varied from 1 to 10 mm. For all these variables, the variation coefficient C_v was within the range 0.1–1.5.

Erosion rate increases with flow velocity. This increase is complicated and cannot be described with an often-used simple power-law function $E \sim U_m^n$, which is a straight line on the log-log

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plot. Calculations show that when velocities are relatively low, erosion rate increases more rapidly than in relatively high velocities.

The detachment rate is controlled by resistance factors: soil cohesion, aggregate size, and soil integrity. The rate of erosion is lower for soils with higher cohesion, this effect being quite obvious. The relationship between erosion rate and soil cohesion is non-linear on a log-log plot, and also could not be expressed with the simple power-law functions. The spatial variability of cohesion has a significant influence on the rate of erosion, which increases with the increase of the variability coefficient. The rate of this increase is low for lower cohesion, where variability of soil cohesion is not an important factor of the detachment rate, and is rather high for higher cohesion values.

The effect of soil integrity on the erosion rate is similar to that of soil cohesion: the rate of erosion decreases with the increase of soil integrity. The erosion rate also increases with the increase of the variability of soil integrity, an effect that is more pronounced for higher soil integrity. These relationships are generally similar to those established for cohesion, although in this case the curves are closer to power-law functions.

The effect of soil aggregate size on the erosion rate is quite simple: erosion rate decreases with the size of soil aggregates and increases with its variability. The effect of aggregate size is more obvious in lower velocities and for soils with lower cohesion and integrity.

Conclusion

A new stochastic method of erosion rate estimation, based on calculation of the probability of excess of driving forces above resistance forces in the interaction of water flow and structured soil, was used in erosion modelling. Knowledge of the probability density functions for flow velocity, soil cohesion, aggregate size and soil integrity makes it possible to calculate theoretically the erosion rate of cohesive soil for any combination of these stochastic variables. The proposed theory allows the explicit explanation of the variability in relationships between detachment rate and flow velocity. The understanding of the stochastic mechanics of soil erosion opens the possibility of theoretically finding new consistent patterns of the phenomenon, which were previously known only from empirical observations and were difficult to explain. These new capacities of stochastic soil erosion models make them the models of the third generation, because they involve new techniques and provide significant new information compared with the first-generation statistical USLE-type and the second-generation process-based deterministic shear stress-type models.

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